

# THE MEDIEVAL ‘SCIENTIA’ OF STRUCTURES: THE RULES OF RODRIGO GIL DE HONTAÑÓN

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## INTRODUCTION

The building of Gothic churches and cathedrals was not an amateur task. Medieval builders were “masters”. Gothic structures justify this title and even today with a well developed structural theory very few architects or engineers, if any, would dare to “sign” similar projects (and this is a problem in restoration work and structural expertises).

The science of Statics was not sufficiently developed in the Middle Ages to allow a scientific structural design; in fact scientific structural theory originated in the 17th century (Galileo, Hooke), but began to be applied only in the second half of the 18th century. How is it possible, then, that the Gothic masters built such magnificent structures? Was the design of Gothic cathedrals a matter of pure chance, the result of a blind trial and error process? Is, therefore, the history of Gothic architecture plagued with collapses and ruined buildings? In fact this was not so. There were collapses, but very few in comparison with the number of successes. Besides, there were so many “mutations”, entirely new types of structures, as to invalidate completely an “evolutionary” theory based on the survival of the more apt designs. The development of Gothic was revolutionary, an explosion of structural creativity.

The Gothic masterbuilders had a *scientia*, a theory, a body of knowledge which permitted to design and build safe structures [Heyman, 1999]. This *scientia* was not “scientific” in the sense we give today to this word; it was not deduced from general laws and scientific principles, it was not an “applied science”. The set of rules and procedures were deduced empirically, from the

observation of existing buildings. This empirical approach is not altogether unscientific. Each building was a “successful experiment” and the observation of ruins and collapses was also very informative. Finally, during the building process the masonry structure moves and shakes, adapting itself to the different phases of construction. These movements suggest corrections to improve the stability of the work.

What was then, precisely, the nature of this medieval *scientia* of structures? This is a difficult question to answer. It must have been a wide and complex body of knowledge. The construction of a Gothic cathedral involved many different operations: surveying, soil mechanics, foundation design, centering, buttress and vault design, stereotomy, carpentry, lifting devices, labour organisation, etc. These are the modern keywords for some of the activities involved. The master of the work had to make decisions in all these aspects which were probably intertwined in a complex way. The depth of understanding in all these aspects could be best judged from the results. Consider, for example, Beauvais cathedral. One can feel a security of design, an absence of doubts, a determination, which could arise only from a mastery of the building processes.

Buildings are, then, our primary source and any hypothesis concerning the nature of the medieval *scientia* of structures must account for the evidence of so many churches and cathedrals which have survived during centuries. Literary sources from the Gothic period are scarce [Frankl, 1960] and only very few Gothic manuscripts about building design have survived, most of them from the late-Gothic period. Not very much information to infer the nature of a knowledge which, as has been already said, was rich and complex.

Only the Album of Villard de Honnecourt pertains to the classic Gothic era, the age of wonder when the “best” cathedrals were built. But Villard is silent on structural matters. No concrete structural rules or observation could be found in his Album. Besides, he was probably no expert in structural matters as demonstrates the wrong position of the flying buttress in the section of Reims cathedral [Hahnloser, 1972]. They are placed too low, at the level of the vault springings; an error which no master would have committed (maybe he copied a drawing from the lodge).

On the other hand, a lot of information on structural matters can be found in the some late-Gothic manuscripts. Some of them could be called “Treatises”; they contain information in all aspects involved in the design of a Gothic church. Others treat only particular aspects: the design of gablets or pinnacles, or the solution of certain geometrical problems [Shelby, 1977]. Finally, some expertises concerning structural problems have also survived and they are an invaluable source to understand Gothic structural thinking. The expertises of Milan [Ackerman, 1944] or Gerona [Huerta, 1998] has been analysed many times; nevertheless, many documents remain unpublished or unnoticed. For example, in Segovia’s cathedral a discussion aroused on the order

of construction of vaults, walls and flying-buttresses and expertises were written by some of the best contemporary architects, Enrique Egas, Francisco de Colonia and Rodrigo Gil de Hontañón (a diplomatic transcription in Cortón, 1997).

In all the above cited sources is evident an obsession for geometry, for the right measures and proportions. This is not new, and registers of the detailed dimensions of buildings can be traced back to the 6th century B.C. in the Book of Ezequiel [Frankl, 1960: 139; Heyman, 1992]. What is new is the conscientious identification of structural elements and their design as separate entities.

## MEDIEVAL STRUCTURAL RULES

The structural knowledge was codified in the form of rules or receipts [Huerta, 1990]. There were rules to obtain, for example, the size of buttresses or the cross sections of the ribs. These rules were a mere register of right dimensions for different structural elements. By its very nature they are specific and pertain to certain structural types. The application of Gothic rules to a Renaissance building, for example, could lead to disaster. The thrust of a Gothic cross vault could be less than one half the thrust of a Renaissance barrel vault. Periods of transition were critical and, indeed, there is documentary evidence both in treatises and in the registers of many churches of pathologies associated to the use of the wrong rules [Huerta, 1997].

In this paper only some specific structural rules are investigated, particularly those rules for vault and buttress design, with some comments also on tower design. We are going to consider, then, only one aspect of the whole process of vault design and construction. This separation is arbitrary; building is not the sum of several independent activities.

### *Late-Gothic German rules*

Coenen [Coenen, 1990] has published a diplomatic edition of the main late-Gothic German treatises ("Werkmeisterbücher"), all written during the 15th century. Three of them are true architectural treatises and contemplate the whole process of church design: The *Unterweisungen* (Instructions) by Lorenz Lechler, *Von des Chores Maß und Gerechtigkeit* and the *Wiener Werkmeisterbuch*. Coenen has made an analysis of their contents and Shelby and Mark [Shelby & Mark, 1979] have studied the structural aspects in Lechler's treatise. However, a critical edition is lacking and they are difficult to read and interpret. The main structural rules refer to the design of buttresses, vaults and towers.

### Buttresses

A buttress should have a depth  $d$  three times the thickness  $t$  of the wall, which is one-tenth of the span  $s$  ( $t = s/10$ ); the buttress' breadth is equal to the wall thickness. This leads to a dimension  $d = 3t = s/3.33$  (at the base); this basic dimension could be diminished or increased depending on the quality, good or bad, of the masonry. Also, the cross section diminishes in height with taluses. The rule is cited several times in all the three treatises; the proportions could be found in many churches of this period and, also, in many of the surviving plans (Fig. 1).

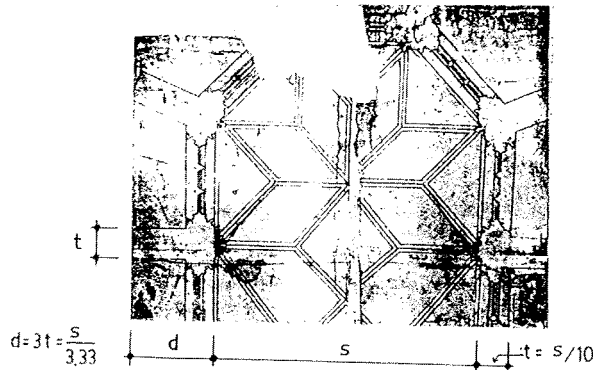


Fig. 1. Proportions of wall and buttresses measured in a Gothic "riss" [Koeopf, 1969: abb. 38].

### Vaults

A Gothic vault is composed of ribs, keystones and webs (curved masonry that fills the voids between ribs). Only the ribs are mentioned. It is said specifically that the cross ribs are semicircular; other instructions referring to the geometry of the other ribs are difficult to interpret due to the absence of drawings [Shelby & Mark, 1979]. There are several rules for the transverse sections of the ribs, some of them contradictory. As an example, Lechler says that the depth of the cross rib should be one-third of the the wall thickness (that is  $s/30$ ), and the width was to be one-half of its depth. The dimensions of the transverse ribs were function of the cross-rib. Transverse arches should be one-third larger than the cross-ribs (span/22 nearly).

### Towers

High towers surmounted with spires are as typical of Gothic architecture as flying buttresses and cross vaults. The relevant parameter, given the plan and general proportion of the tower (relation between the side and the height), is the wall thickness. Two of the treatises gave the same rule: the wall thickness of the tower should be  $1/20$  of its height [Coenen, 1990: 99]. If the tower has

counterforts these were to have the same depth as the wall thickness and a breadth two-thirds the wall thickness [Coenen, 1990: 104]. The first rule for the wall thickness must have been a common rule in Germany because Albrecht Dürer (1525) used it in his *Unterweisung der Messung* (Geometrical instructions) when he explains the design of a city tower of 300 feet of height (Fig. 2).

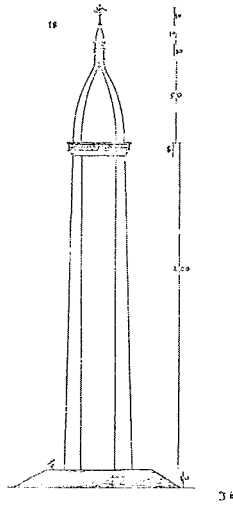


Fig. 2. Design for a city tower [Dürer, 1525: abb. 18].

### *Geometrical rules for Gothic buttresses*

Other Gothic rules have survived through Renaissance or Baroque treatises of stereotomy or architecture. Two of them are important for their diffusion. Both rules refer to the dimensioning of Gothic buttresses.

#### *Geometrical rule n° 1*

The rule permits to obtain the buttress for a cross vault using the profile of the transverse arches. It was published for the first time by Derand (1643) in his *Architecture des Voûtes*. But the rule can be traced back, at least, to the first half of the 16th century in Germany [Müller, 1990: 237]. It appears again, in the second half of this same century in the unpublished manuscript on stonecutting of Martínez de Aranda (ca. 1590) [Bonet, 1986]. The rule was well known in the 17th century. It was cited in the popular *Traité d'architecture* of J. F. Blondel (1694). During the 18th and 19th century appeared in many treatises: Wren (1750) cites it and for Vittone it is the most safe of all the rules [Benvenuto, 1981: 323]. Viollet-le-Duc [1874: 4, 63] gave also the rule. Barberot [Barberot, 1895] said it is a good rule to size the buttresses of simple arches and even

some manuals of vault construction of the second half of the 20th century cite it [Cassinello, 1964].

Though since the beginning of the 18th century is applied to size the buttresses of simple arches, it is a Gothic rule and applies to Gothic buttresses. Derand (1643) said this explicitly and Ungewitter [Ungewitter, 1859] and Ungewitter and Mohrmann [Ungewitter & Mohrmann, 1890] used it in this context applying the rule to the transverse arch of the Gothic vaults.

The rule is as follows: in Figure 2 (a) from Derand the arc LI is divided in three equal parts by the points (N) and M. The line MI is then prolonged so that  $MI = I(A)$ . The point (A) defines the outer edge of the buttress (points (A) and (N) has been added to help to explain the rule). In Figure 2 (b), from Martínez de Aranda the same dimension is obtained by an alternative, more simple, construction. Again the arc is divided in three equal parts by two points. Trace a perpendicular from one of them, *g*, to the springing line to obtain point *h*. The distance *ih* is the thickness of the buttress. When applied to some single nave Gothic buildings the rule gives good concordance (Fig. 3). Warth [Warth, 1903] showed that it could be applied also to the design of the external abutments that support the flying buttresses.

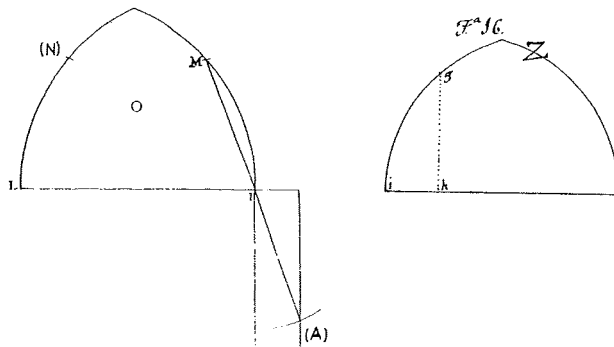


Fig. 3. Geometrical rule n°1: left, Derand (1643); right, Martínez de Aranda (ca. 1590).

### *Geometrical rule n° 2*

In the unpublished *Architectural Treatise* of Hernán Ruiz el Joven, a Spanish architect of the 16th century [Navascués, 1974], we find another rule for buttresses of Gothic origin. This rule has remained till now unnoticed [Huerta, 1990]. Hernán Ruiz gives the rule as a method to obtain the abutment for simple arches, but it is, again, a Gothic rule for buttress design. The same construction is given by Ungewitter [Ungewitter, 1859: 281] as a rule to size the buttresses of a polygonal Gothic apse. Ungewitter says nothing of its origin, but it is very probable that both have the same Gothic origin. The rule could be found, again, in England. Apparently Betty

Langley used the same construction to size the piles of Westminster Bridge [Huerta, 1990: 144]. The appearance of the same rule in so different places and epochs is a demonstration of their importance and diffusion.

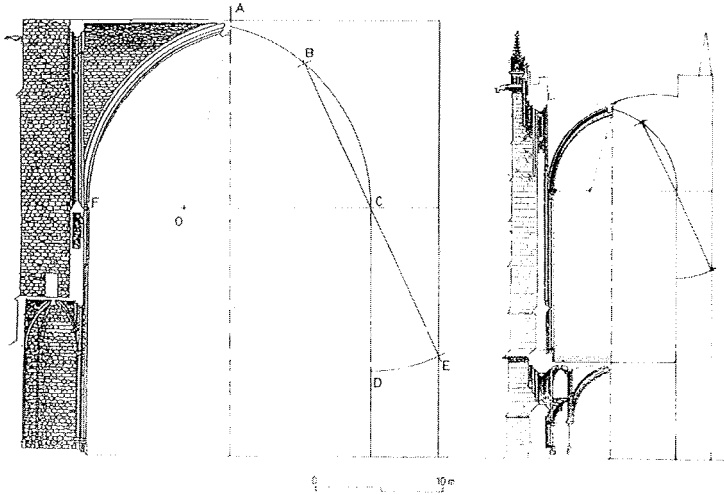


Fig. 4. Application of geometrical rule n°1: left, Gerona's cathedral; right, Sainte Chapelle.

The rule is as follows: consider a drawing of half the transverse arch of a Gothic vault with its thickness. Draw the chord of the semi-arch, then trace a parallel line tangent to the extrados; the point where this line cuts the horizontal line of the arch springings defines the thickness of the buttress. The results are very similar to those obtained with the previous rule.

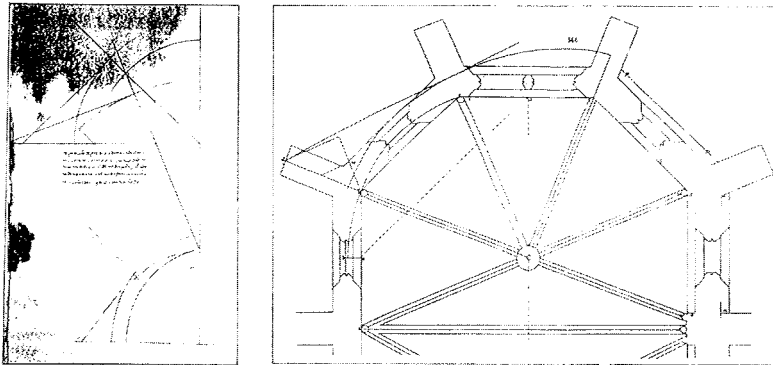


Fig. 5. Geometrical rule n° 2: left, manuscript of Hernán Ruiz; right, Ungewitter (1859).

## RODRIGO GIL DE HONTAÑÓN

### *The treatise*

Rodrigo Gil (1500-1577) was maybe the most important and prolific Spanish architect of the 16th century. The son of a famous Gothic masterbuilder, Juan Gil de Hontañón, he inherited the tradition of Gothic construction, but during his life he assimilated also the new vocabulary of the Renaissance. He participated to a greater or lesser degree in the construction of nine cathedrals (Astorga, Salamanca, Segovia, Plasencia, Santiago, etc.) and built many parish churches and civil buildings. Between 1544-1554 wrote a Treatise of Architecture [Sanabria, 1982: 283] which was copied by Simón García in his *Compendio de Arquitectura* written 1681-1685, forming the first six chapters. Chapter 75 and an illustration at the end of chapter 16 are also attributed to him. In this work he tried to reconcile Gothic design methods (“por iometria”, based in geometry) with Renaissance methods (“por analogía”, by analogy with the human body). There are two facsimile editions and an English translation by Sanabria (1984) in volume II of his PhD. In this paper all the English quotations to the manuscript in this paper are Sanabria’s translation. References to the manuscript are in square brackets.

The manuscript treats in a systematic way the different aspects of the design of a late-Gothic hall church. First the surface is calculated in function of the parish population; then the general form of the plan, number of naves and the proportions of the nave bays. After that he discusses the heights of the naves, the design of towers and spiral staircases, etc. Finally, in chapter 6, he treats specifically the sizing of structural elements using certain general rules (“reglas generales” says Rodrigo). It is this last part which converts the manuscript in something unique. In no other Gothic source appears such a conscientious separation of the structural skeleton. The structural rules are completely independent of the general design and constitute the first documented trial to create an independent science of structures. In spite of this, the rules have not received great attention: only Kubler [Kubler, 1944] and Sanabria [Sanabria, 1982, 1984, 1989] have studied them in detail.

The rules could be divided in two groups:

- 1) rules for the design of the structural elements of a Gothic church;
- 2) rules to investigate the buttress for a simple arch or barrel vault.

It is important to make this distinction which is justified by their location in the manuscript and, above all, by their different goals: practical in the first case, of research in the second.



### Rules for church design

In the 16th century most of the churches built in Spain were hall churches of three naves of the same height covered by a special type of Gothic vault, the “bóvedas baídas”. These vaults are of domical form and the ribs are very nearly disposed in the surface of a sphere which has as diameter the diagonal of the bay (cross ribs are perfect semicircles). All the examples in the manuscript correspond to this type of vault.

Rodrigo says explicitly that he prefers this disposition and argues that it is better for the equilibrium of the building: “...having one height for all aisles strengthens the building because all parts support one another, which is not the case when the central one is taller.” (Chap.3, 8v). Ahead in the manuscript he insists on this aspect: “The buttress not only sustains the transverse arch of the outer side aisle or chapel, but also that of the inner side aisle and of the nave. If these naves are constructed to the same height their transverse arches reinforce each other, but if the side aisle is lower, the pier sustaining them must be made thicker” (Chap.6, 21v).

Rodrigo explains, then, the process of construction of the vaults, Figure 6 left, but he notes that “...these things may be difficult to understand if one lacks experience and practice, or if one is not a stone mason, or has never been present at the closing of a rib vault, so that one has not become competent in the laying of ribs and liernes.” (Chap.6, 24r)

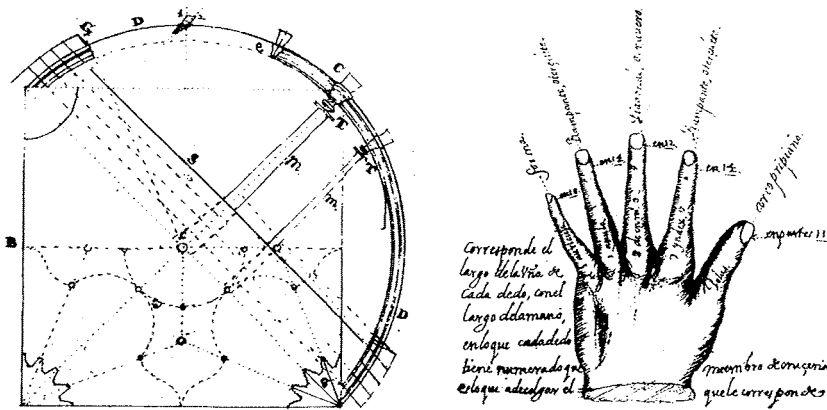


Fig. 6. Right: construction of a cross vault. Left: sizing of the ribs by analogy with the hand.

First, a platform is built at level of the tas-de-charge (a little above of the springings). There the plan of the vault is drawn and the keystones are placed in position above wooden struts. Then, centerings between the keystones were constructed, the ribs were built and finally the masonry web between the ribs was laid. The rib skeleton function as a permanent centering and ribs and

keystones should have certain dimensions so that this skeleton would be stable, not only at the end, but during the whole building process.

After defining the general proportions of the church, Rodrigo exposes his general rules: “Since we have dealt with the distribution and all its intervals, it will be good to deal with the thickness of piers and the projections of the buttresses so that all (parts) may be measured and proportioned.” (Chap.6, 17r)

### *Piers*

Rodrigo gives a rule to obtain the diameter (piers were usually cylindrical) of the interior piers of hall churches. The rule is arithmetical and contains a square root but it is exposed discursively, by writing: “Returning to the thickness of the piers, I say that the width of a nave bay, 40 feet, should be added to the length, 30, which is 70. To this should be added the height of the column, 40 feet, which is 110. The square root of 110 is 10-10/21, half of this is 5-5/21, and this should be the diameter of the column on the lower part. This is the closest to what is right.” (Chap.6, 17r). The rule can be expressed algebraically (Sanabria 1982: 286):

$$D = \frac{1}{2} \sqrt{h + w + l} \quad (1)$$

where,  $h$  is the height of the pier, and  $w$  and  $l$  are the width and length of that bay. The rule is not dimensionally correct and to obtain good results the data should be in Castilian feet (0.28 m); if we introduce the dimensions in meters the results are multiplied nearly by a factor of two.

Sanabria committed this error trying to verify the application of the rule to the piers of the cathedral of Barbastro:  $h = 21$  m;  $w = l = 10.5$  m (square bays). If we enter the dimensions in meters the rule gives a diameter of 3.2 m; if we enter in feet we obtain 6.1 feet or 1.7 m. The actual piers have a diameter of 1.3 m. Applying, again, the rule to the Colegiata of Lerma in Burgos ( $h = 11$  m;  $w = 6,7$ ;  $l = 6.5$ ) we obtain 4.7 feet or 1.3 m; Sanabria obtained 2.5 m and the actual piers have a diameter of 1.76 m. So in the first case Rodrigo’s rule gives a thicker pier and in the second a more slender one. In any case the agreement is fairly good. Sanabria’s trivial error leads him to conclude that: “the formula is very conservative, and has a substantial safety factor built-in.”

### *Buttresses*

Another arithmetical rule is given to determine the size of the vault buttresses. Rodrigo gives first the rule and then applies it to a vault of certain dimensions. It is an important rule and he wanted, possibly, that no error could be committed. The text says (following Sanabria’s translation):

“To find the necessary projection of the pier buttress, add up the feet of circumference (i.e., the perimeter) of the ribs supported by the buttress. By this is to be understood half of the length of the ribs, which is the lengths of the tiercerons to their keystones, the lengths of the diagonal ribs to their central bosses and half of the length of the transverse arch. Having added up all this, subtract one third, which is what is normally taken up by the mouldings. Should the mouldings take up more or less subtract more or less accordingly. Now measure the height of the buttress, and add it to the remainder of the previous operation. Take the square root, and divide it by three. One of these thirds will be the width of the buttress, and the remaining two thirds its length, including the engaged half column, the wall thickness, and the external projection” (Chap.6, 17v). Algebraically:

$$C = \frac{2}{3} \sqrt{h + \frac{2}{3} \sum Ri} \quad (2)$$

where  $C$  is the total thickness of the buttress (including the wall) at the level of the springings of the vault,  $h$  is the height of the buttress and  $\sum Ri$  is the sum of the lengths of the ribs converging on the buttress, measured from the springing to their respective keystones. The breadth of the buttress is  $C/2$ .

After giving a detailed numerical example Rodrigo affirms: “This is the right size to hold the thrust of the arches. The workman can add somewhat more, because it is better to have too much than too little, although this size will be sufficient, as was stated.” (Chap.6, 18r)

The rule is cited again twice in other parts of the manuscript. The first time at the beginning of Chapter 2 where he discusses several church designs, here he applies simply the rule without explanation, as a routine calculation (Cap.2, fol. 5r.). It appears again at the end of chapter 6 where Rodrigo remarks strongly the goodness of the rule: “Thus seeking the *intrinsic reasons and irreproachable causes*, it is necessary first to study the elevation of the temple to determine which members are thrusting against the buttress...Having followed all the various instructions discussed above *the result will be strong, safe, beautiful, and proper.*” (Chap.6, 22r, 22v. My italics).

If we are to believe Rodrigo's word he had a great confidence in the rule, which was not a mere arithmetical experimentation.

#### *Vaults: ribs and keystones*

The sizing of ribs and keystones is treated together. Rodrigo stresses the importance of the problem: “It is good to know the correct size and thickness of the ribs and bosses of rib vaults, since we have seen many ruined either because their bosses were too heavy and thus much larger

than what the ribs could hold, or else much too light so that the weight of the ribs lift them causing cracks to open in the walls.” (Chap.6, 22v) Rodrigo alludes, probably, not only to the completed vault but, also, to the vault under construction.

For the ribs he gives simple arithmetical formulae. It is interesting that he tries to reconcile older Gothic geometrical rules with the design by analogy with the human body. In this case he uses the hand: “Now in order to have a general rule, which is what we want, we must understand that the thumb may be viewed as the transverse arch, the index and ring fingers as tiercerons, the middle finger as the diagonal rib, and the little finger as the formeret. To determine the proportions of the fingers to the hands, take half the ounces of these fingers, which is the length of each fingernail... dividing the length, or side, or a bay in 20 parts, one part shall be the height of the voussours of the transverse rib. The length of the bay divided in 24 parts shall be the height of the diagonal rib. The tiercerons will be one twenty eighth, and the formeret one thirtieth. Thus shall they be proportioned, in accordance with the work they do. (Chap.6, 23r)

The thickness of the ribs in function of the span  $s$  are:

- transverse ribs	$s/20$
- cross ribs	$s/24$
- tiercerons	$s/28$
- formerets	$s/30$

This is when the height of the vault is equal to the span and the bay is a square. If the height is different Rodrigo says that the ribs should be made thicker or thinner in the same proportion: “Note that we give this rule assuming that the bay elevation to the capitals is equal to its side. If the elevation should be greater or smaller, add or subtract using rule of three.” When it has a rectangular form: “If the bay should be oblong do not take either the long or the short sides but add them and divide by two.” (Chap.6, 23v). Being a practical rule that will be often used, Rodrigo explains all the possible cases.

For the keystones the rule is again arithmetical. It is one of the most difficult rules to interpret. The rule gives the weight of the keystones in “quintales” (a quintal = 46 kg or, approximately, the weight of a cubic foot of a medium stone). In the formula enter again the lengths of the ribs, but a distinction should be made between those members that “sustain” and those that “are sustained”: “Those that are sustained must be subtracted from those that sustain. They can be told apart because those that sustain spring from the tas-de-charge, and those that are sustained spring from bosses. There are also sustaining and sustained bosses. Those found along the lengths of the diagonal rib or tiercerons are sustained. Those that are on the ends of the diagonal ribs or tiercerons sustain all others.” (Chap.6, 23v, 24r) Then Rodrigo gives his formula, which can be written algebraically:

$$C = P\sqrt{\sum Ri - \sum Si} \tag{3}$$

where  $Q$  = weight of the boss in quintales;  $P$  = weight of the cross rib (quintales/foot);  $\sum Ri$ =sum of the lengths of the ribs that sustain;  $\sum Si$ =sum of the lengths of the ribs that are sustained. The rule is, again, dimensionally incorrect. To use the rule correctly we should enter the data in Castilian feet and quintales, and the result will be in quintales. If we enter with usual dimensions we obtain a weight which is of the order of the weights of actual vaults (the rule admits several interpretations and these results should be taken with care). However the existence of the rule stresses the importance which keystones have for Rodrigo as structural elements. The keystones serve, obviously, to solve a complicated steretomic problem (the union of different ribs), but they play also a fundamental role stabilizing the rib skeleton during the construction of the masonry webs. This possible structural role of the heavy Gothic keystones has been practically unnoticed.

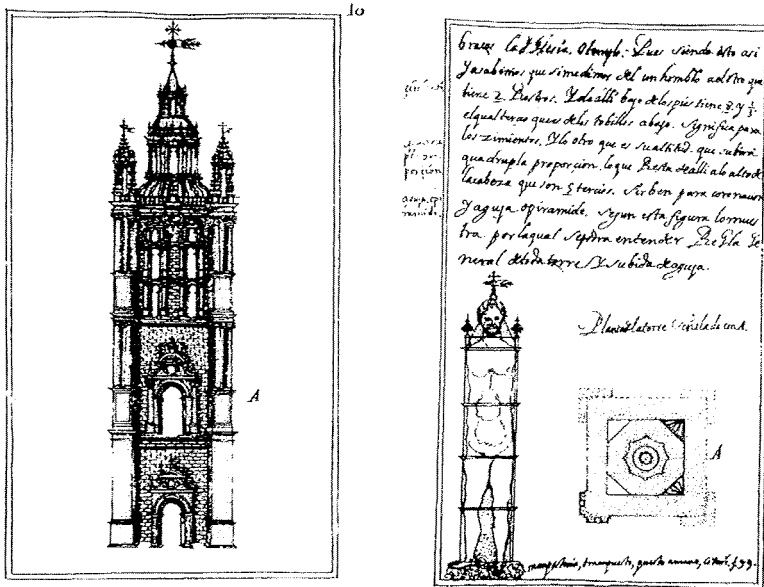


Fig. 7. Design of towers in the Treatise of Rodrigo Gil de Hontañón.

### Towers

Rodrigo treats also the structural design of towers. First he discusses the general proportions of the tower using the analogy with the human body (Fig. 7). Then, he gives rules to size the wall thickness and the counterforts of the towers. The rules are arithmetical and could be written algebraically:

$$W = \frac{1}{2}\sqrt{H} \quad (4)$$

$$B = \frac{1}{2}\sqrt{H+A} \quad (5)$$

where  $W$  is the wall thickness and  $B$  the buttress thickness at the top of the tower;  $H$ =height of the tower;  $A$  = length of the side of the base.

Rodrigo also alludes to a modification suggested by “Other expert architects and arithmeticians” which consists in adding to the above mentioned height “half the circumference of the semidome that vaults the uppermost tower chamber... and this dimension is one of the most fully attained.” (Chap.2, 5v, 6r).

It is most interesting this allusion to other rules by other architects. No doubt Rodrigo would have known also the detailed proportional rules given by Alberti who recommends as wall thickness one-fifteenth of the tower height [Huerta, 1990: 155].

In the manuscript we find evidence of the practical application of the rules. Chapter 75 of García’s *Compendio* has the title “General conditions to rebuild a ruined building”. The ruined building in question is a tower and the text is a report written by Rodrigo describing carefully the demolition of the ruin and the construction of a new tower (the elevation in Figure 7, left). The tower was to have a height of 120 feet. Rodrigo does not cite any rule but recommends as wall thickness 5 feet and as buttress thickness 7 feet. If we use his rules the wall thickness should be 5.5 feet and the buttress (for a side of 30 feet) 6.1 feet. There is no doubt that Rodrigo is using his rules in the structural design of the new tower.

### ***Rules for simple arches and barrel vaults***

Rodrigo manifest no doubts in designing Gothic vaults, buttresses and towers. His rules were probably an empirical adjustment of the data of many buildings, data which he would have inherited from his father. But when it comes to design the buttress for a single arch or barrel vault, Rodrigo confesses himself at a loss. He commences the corresponding section saying:

“I have tried many times to account for the buttress that any arch may need, but I have never found any rule to be sufficient. I have also discussed this with both Spanish and foreign architects, and none seems to have been able to verify such a rule: but all follow their own judgement. When I ask how do we know that so much is sufficient for a buttress, the answer is that it needs that much, but no reason is given” (Chap.6, 18v, 19r).

The word “reason” here does not refer to a certain scientific theory; reason, (“razón”) in Spanish means also “the order and method to do something”. Rodrigo wanted a set of verified

procedures, like those he used in the design of Gothic structures. A simple barrel vault was an alien structure to him (as far as I know he built none) and he was perplexed.

The section, then, has an experimental nature. Rodrigo gives four different geometrical rules and an arithmetical rule. There is no space here to discuss the types and evolution of the rules [Sanabria, 1982, Huerta, 1990] but their experimental character is evident. Sanabria has even suggested that the rules may be a register of actual experiments with real arches, and there are many arguments in favour of this hypothesis. In any case, it is evident that Rodrigo knew the specific character of the Gothic rules and he does not even try to apply them to the new structural type.

## CONCLUSION: VALIDITY OF THE RULES

As we have seen the Gothic masterbuilders used empirical rules for the design of the structural elements of their buildings. These rules had a great diffusion, geographical and chronological, and there is abundant evidence of its use throughout Europe.

### *Proportional rules*

A great majority of the structural rules for masonry are “proportional”, that is to say, they produce “similar” forms in a geometrical sense. They give, for example, the depth of the buttress for an arch depending on its curve of intrados but regardless of its size. In other words, they implicitly believe in the existence of a “law of similitude”: a valid structural form continues to be correct independently of its size (Fig. 4).

Galileo argued the impossibility of the existence of this kind of principle: in structures supporting as the main load their own weight the dead load rises as the cube of the linear dimensions while the section of the structural members rise as the square; the tensions rise, therefore, linearly with the size. Galileo's argument has achieved the rank of law, the “square-cube law”, in structural design and it has determined the attitude of engineers and architects to the effects of scaling in structural design, and of building and civil engineering historians towards the traditional proportional rules which has been considered usually as unscientific and, therefore, incorrect.

Galileo's argument is valid only when the criterion of strength governs the design. As has pointed many times Professor Heyman [Heyman, 1995b, 1996] this is not the case with masonry structures: the most restricted condition is that of stability. A masonry structure will be safe if it is possible to find a system of compressive internal forces in equilibrium with the loads. The

thrust lines should be contained within the masonry. This is a geometrical condition, which depends on the form of the structure but not on its size. Proportional rules are therefore of the correct form and the old masterbuilders possessed this all-important knowledge. Of course the strength imposes some limits on size but such limits are very far from the dimensions of even the greatest masonry structures [Huerta, 1989, 1990].

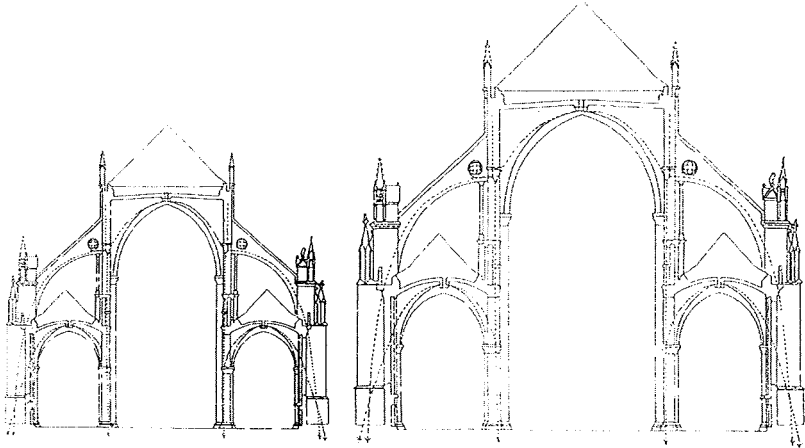


Fig. 8. Scaling up and down does not affect the safety of a masonry structure.

### *Non-proportional rules*

Many of the rules of Rodrigo Gil de Hontañón are non-proportional, not even dimensionally correct. They have been considered therefore incorrect and nonsensical. In fact they refer to non-proportional problems. The thickness of the webs of a Gothic vault, for example, is very often constant: the minimum that can be practically built (0.15-0.20 m of stone). In this situation, the weight, and therefore the thrust, of a Gothic vault rises with the square of its linear dimensions; the weight of the buttresses on the contrary rises with the cube. If we scale up a building it will need buttresses proportionally more slender. Just the contrary of Galileo's square-cube law. The same occurs with high towers. Here the main load is the action of wind. The total thrust of the wind rises with the cross sectional surface of the tower, but its weight grows with its volume. Again, greater towers could have proportionally lesser thickness, and this property could easily be seen if we compare similar towers of different sizes (Fig. 8).

Finally, Rodrigo Gil stressed the importance of a correct size for the heavy Gothic keystones. The skeleton of ribs must be stable during construction. Arch rib design is proportional and the rules are a fraction of the span. Web construction would have proceeded



from the perimeter to the centre of the bay. In this situation it is possible that an arch charged mainly in the haunches could collapse by rising of its central keystone. Keystones placed on top of wooden struts were a "passive weight" which was "used", if necessary, to stabilize the rib skeleton during construction. They have the same structural function as the heavy keystones for pointed arches (Fig. 9).

Non-proportional rules represent a finer adjustment, to the involved variables. They were deduced empirically, from a close observation of existing structures or of structures under construction. What is important is that they point to important design aspects. An unprejudiced critical reading of the old Gothic treatises has served to disclose some properties of the design of masonry structures that are usually not noticed.

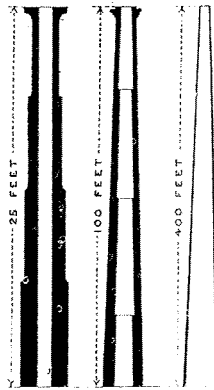


Fig. 8. "Inverse" law in tower design [Barr, 1899].

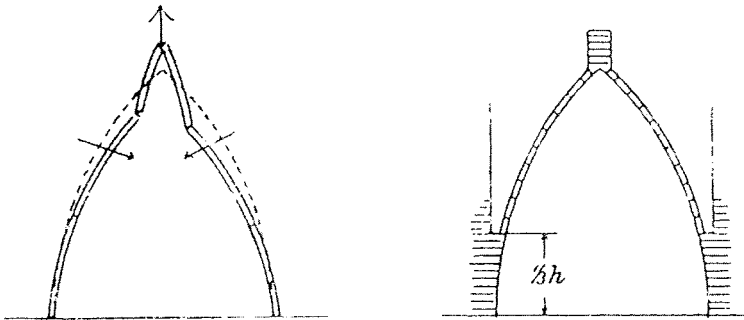


Fig. 9. Stabilizing function of keystones for pointed arches [Ungewitter, 1890].

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